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# Comparison of stress concentration versus minimum solid area based mechanical property–porosity relations

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Stress concentrations due to pore shape are questioned as a fundamental determinant of mechanical property–porosity relations, especially elastic property–porosity relations. On the other hand, actual solid load-bearing areas, especially minimum solid areas of porous bodies, clearly are a determinant of mechanical property–porosity effects. The correlation of pore shape–stress concentration effects with elastic properties of ceramics can be explained by the correlation of pore shapes with minimum solid areas.

## 1. Introduction

Mechanical property–porosity relations are generally based on one of three types of approach. One geometrically derives the actual load-bearing areas, neglecting any stress concentration or related pore shape effects other than how they effect the actual load-bearing area [1–9]. While some simpler load-bearing models use the average solid cross-sectional area, the minimum solid cross-sectional area supporting the load is more appropriate. For the two basic ways of forming porous bodies, namely partial sintering of particles or introducing larger voids (e.g. via bubbles or fugitive particles), minimum solid (load-bearing) areas are respectively the actual sintered or bond areas between particles and the minimum web cross-sections between pores. While such areas are most readily calculated for idealized structure, i.e. regular stackings of pores or particles of uniform size and shape [1–8], such concepts are readily extended to other bodies, e.g. via use of average particle sizes and shapes or combinations of idealized structures [1–8].

A second approach, again based on idealized geometries and their generalization, is based on pore shape and the resulting stress concentration [1, 10, 11], often the maximum concentration [1, 12]. A third approach is to instead apply more general mechanical (e.g. strain) analysis to bodies of some limited but generalized geometry, e.g. where pores are "in the mean" "spherical", "uniformly distributed", etc. [1, 11]. Such analysis may implicitly or explicitly involve stress concentrations. Both shape–stress concentration and minimum solid area concepts are normally and most easily applied to idealized regular arrays of identical size and oriented (for non-spherical) particles, pores, or both, and should be applicable to non-idealized cases by modern computational means. However a fundamental question is the applicability of such shape–stress concentration concepts to the porosity dependence of properties. The solid load-bearing area clearly plays a basic role in not only mechanical but all physical properties [1]. Stress concentrations also clearly play a basic role in mechanical properties. However, it has been assumed [1, 11, 12], with little or no justification, that stress concentration effects associated with cracks and other macroscopic stress raisers, applied on a microscopic level, determine a variety of mechanical, including elastic, behaviours. Also, while stress concentration concepts are generally not applicable to most other physical properties, pore shape concepts have been applied to the porosity dependence of other properties.

A detailed examination of stress concentration–pore shape effects would be a large subject. This note instead presents basic observations showing that microstructural stress concentrations, e.g. from pores, generally play at best a limited role in determining mechanical properties. It further shows both specific differences and correlations for pore shape–stress concentration and minimum solid area and crack models for mechanical property–porosity data. Two other papers address pore shape and minimum solid area evaluations [4, 13].

## 2. Pertinence of pore shape–stress concentration effects to mechanical property–porosity relations

Consider first elastic properties. Both homogeneous and heterogeneous, e.g. porous, bodies follow the linear elastic stress–strain behaviour of Hooke's law. However, porous bodies differ in that (i) the slopes of their stress–strain curves (i.e. their elastic moduli) are lower than for corresponding dense (homogeneous) bodies, and (ii) their macro- and micro-stresses and strains are not the same as they are in homogeneous bodies. Porous bodies have varying stresses and strains whose magnitudes are determined by pore stress concentrations and Hooke's law (which holds to significant fractions of the theoretical strengths of brittle materials). However, since the net body strain is an average of the local strains reflecting both the pore shape and orientation (for non-spherical pores), the maximum stress concentration cannot be a major determinate of the body strain. On the other hand, average stress concentrations must be ~ 0, and hence not a basic determinant of body strains since they consist of both tensile and compressive components (i.e. opposite sign strains) making averages low or zero. Changing between tensile and compressive body loading switches the spatial locations of tensile and compressive strains from pores within the body, so if they did not average to zero, there would be different elastic responses for compressive and tensile loading. On the other hand, stress-strain (and hence modulus) behaviour clearly correlates with minimum solid (load-bearing) areas [1–5, 9, 13].

That microstructural stress concentrations have no intrinsic effect on elastic properties is shown by considering the effects of stresses between grains. All crystalline materials, even cubic ones, have varying degrees of elastic anisotropy (EA), often rather substantial, as for example in $CaF_2$, MgO, $ThO_2$, $UO_2$, $BaTiO_3$, $MgAl_2O_4$, SiC and TiC [14]. Thus, applied stresses are heterogeneously distributed in such bodies, with maximum concentrations at grain boundaries depending on the degree of EA and grain misorientations. However, measured polycrystalline elastic properties of cubic materials, e.g. MgO, $ThO_2$, $UO_2$ and $MgAl_2O_4$, agree within about ± 3% with the averages of their single-crystal elastic properties [1], i.e. totally neglecting any microstructural stress concentration effects due to EA.

All non-cubic crystalline materials, in addition to having EA, also have thermal expansion anisotropy (TEA), which adds built-in stresses to those from EA, also concentrated at or near the grain boundaries. Yet again, bodies with TEA stresses show no effect on elastic properties. Measured polycrystalline and averaged single-crystal Young's moduli for $Al_2O_3$, BeO and ZnO again agree within about ± 3% (so long as grain sizes are small enough to avoid microcracking) [1]. Similarly the elastic properties of composites are determined by the properties of the constituents, not the microstructural stresses between them [1, 6, 8]. Thus, neither basic concepts nor broad polycrystalline experience shows any significant direct dependence of elastic properties on microstructural stress concentrations *per se*.

Next consider the porosity dependence of tensile strength ($\sigma$), which is typically quite similar, if not identical, to that for Young's modulus ($E$) [1, 9]. This is not surprising since $\sigma$ depends primarily on $E$, the fracture energy ($\gamma$) and flaw size ($C$), i.e. $\sigma \propto (E\gamma/C)^{1/2}$ and $\gamma$ is known to theoretically be closely related to $E$ and indeed, the porosity dependences of $\gamma$ and $E$ are typically similar if not identical [1, 3, 9]. Thus, since $\sigma$ depends on $(E\gamma)^{1/2}$ it also shows a very similar porosity dependence to that of $E$, unless there are overriding effects due to flaw size, i.e. where individual pores become a major portion of the failure-causing flaws [9]. However, flaw sizes are generally considerably larger than pore sizes in sintered bodies (the source of most data) and hence are generally not affected by pore shape and resultant stress concentrations [1]. Even where pore and flaw sizes begin to be comparable, the direct effect of pore shape and resultant stress concentrations on $\sigma$ is significantly mitigated. In such cases, single pores themselves are typically not the sole sources of failure; instead, a pore, or a collection of a few pores, plus associated cracks are now recognized as the sources of failure [1, 9, 15, 16]. The occurrence of cracks associated with pores reduces the effect of pore shape and associated stress concentration. Further, since tensile strength is a weak-link process, the individual or few pores acting as a source of failure will typically be those that are extreme in dimensions and not necessarily representative of the average pore shape.

The fracture toughness and tensile strength of polycrystalline bodies may be influenced by microstructural stresses. TEA (and possibly EA) can reduce strengths by microcrack generation [1, 17] or contribute to the applied stress to aid failure when the crack size is no longer large in comparison to the microstructure [1, 18]. However, in particulate composites microstructural stresses have been directly or indirectly indicated as a source of toughening, strengthening, or both, i.e. contrary to the common assumptions of weakening from such stresses from pores [1, 19].

Consider next other mechanical properties such as compressive strength, hardness and wear, all of which typically involve substantial compressive loading. Properties involving compressive loading of ceramics indicate greater porosity dependence i.e. $b$ values of $6 \pm 3$ [1] versus $4 \pm 2$ for tensile behaviour in the exponential decrease of properties, $e^{-bP}$, with volume fraction porosity $P$. Whether the greater porosity dependence indicated for compressive loading is due to local plastic deformation or accumulation of cracks associated with pore shape–stress concentrations effects [1], or some combination, is uncertain. However, porous metals, which should have more local plastic deformation due to pore stress concentration, have a very similar, if not identical, mechanical property–porosity dependence to ceramics [1], thus questioning the effects of plastic deformation and indicating the need for substantial further study. Finally, note that the successful models of Gibson and Ashby [20] for elastic, toughness and strength behaviour (both tensile and compressive) of very porous, cellular materials are basically load-bearing models and neglect stress concentrations for most materials, e.g. ceramics.

## 3. Correlation between pore shape–stress concentration and pore shape–minimum bond area effects

In view of concepts and data questioning a basic dependence of mechanical property–porosity dependence on pore shape–stress concentration effects, one can legitimately ask why models based on pore shape–stress concentration effects often correlate with mechanical property–porosity behaviour. However, many pore shape–stress concentration effects can be directly related to pore shape–solid area relations.

This can be most readily seen by examining in more detail the commonly used pore shape–stress concentration model for the dependence of elastic properties on porosity derived by Rossi [12]. He modified Hashin's equation for Young's modulus ($E$) as a function of a low concentration of spherical pores [11]:

$$E = E_0(1 - BP) \qquad (1)$$

where $E_0 = E$ when $P = 0$, and $B$ is a parameter dependent on other factors such as Poisson's ratio ($\nu$). Rossi's modification was based on the observation that the $B$ coefficient of Hashin's model partially reflected the stress concentration associated with spheres. Rossi therefore generalized the $B$ coefficients to reflect the maximum stress concentrations of other spheroidal shapes (Fig. 1).

While Equation 1 and the range of $B$ values for Rossi's model are similar to those of many other stress-based models, this is also true for models based on load-bearing area [1]. For example, since the average volume fraction porosity ($P$) is also the average cross-sectional area of the porosity, the average solid area is simply $1 - P$, i.e. Equation 1 with $B = 1$ for any pore shape. The minimum solid area can be readily calculated for idealized bodies of regularly sized, spaced and oriented pores (as considered by Rossi) for direct relation to $E$ [1, 3–8]. For an ordered array of uniform cylindrical pores oriented parallel with the stress axis (for which the average and minimum solid areas are equal) Equation 1 holds, but with $B = 1$ [5]. For an ordered array of uniform spheroidal pores (Fig. 1) simple geometry of the rectangular cells of solid with a pore that makes up such an array [1–8] gives the minimum solid area (i.e. around the equator of the pore) and hence $E/E_0$ as

$$1 - \frac{\pi}{4}\left(\frac{6P}{4K}\right) = 1 - 1.21\left(\frac{P}{K}\right)^{2/3} = 1 - B'P^{2/3} \qquad (2)$$

where $K = c/a$. Such a calculation can be done for other pore geometries [1–3, 7]. The resultant porosity dependence of the minimum solid area and hence of the related physical properties are commonly closely approximated over most of their range of applicability for a wide range of pore character by an exponential function [1–5, 9], e.g. for Young's modulus

$$E = E_0 e^{-bP} \qquad (3)$$

where $b$ is directly related to particle stacking and hence pore shape [1–4, 13, 16]. For limited porosity levels (as for Rossi's model) this is reasonably approximated by the first term in its series expansion, which is again Equation 1 with $B = b$.

Rossi's $B$ values for Equation 1 can be compared with minimum solid area models as follows. They agree exactly for the extreme of prolate spheroids (i.e. cylindrical pores) oriented parallel with the stress as noted above and shown in Fig. 1. For spherical pores equate $E/E_0$ from Equations 1 and 2, i.e. $1 - BP = 1 - B'P^{2/3}$, giving $B = B'P^{-1/3}$ where $B'$ is thus directly calculable, and hence $B$. Plotting the resultant $B$ values for a reasonable range of porosity, e.g. $P = 0.1$ to 0.4 (since such calculated $B$ values are dependent on $P$, and this approach has no meaning for $P = 0$), gives curves for each $P$ similar to Rossi's (Fig. 1). However, while the curves start off identical to Rossi's for cylindrical pores parallel to the stress axis, they fall significantly below Rossi's curve as the pores become more oblate. The magnitudes of the $B$ values calculated

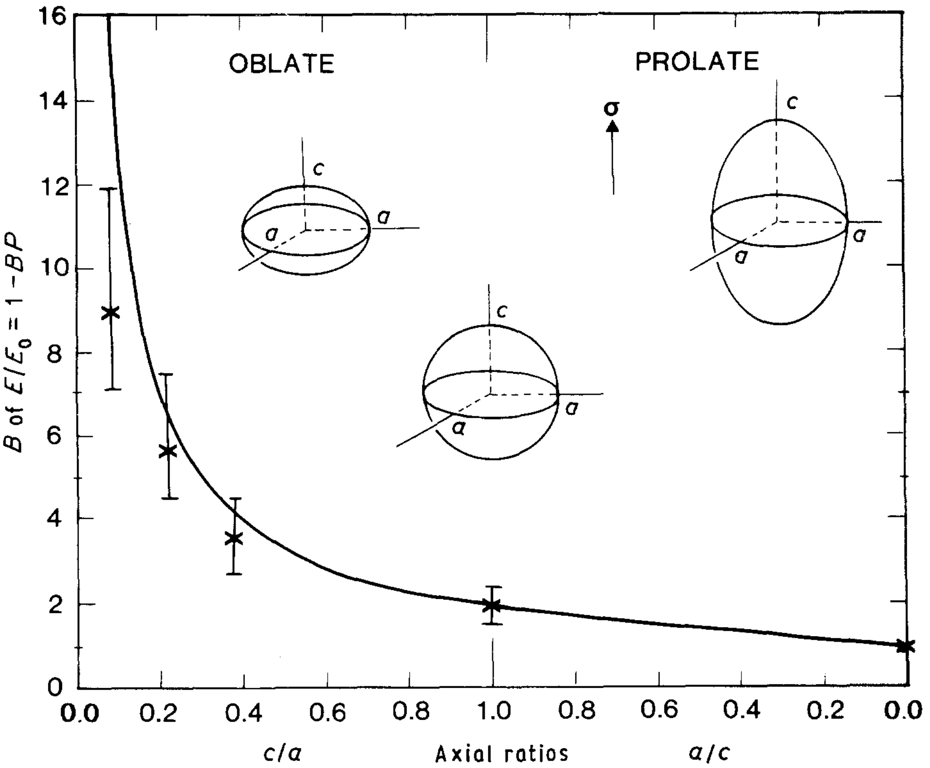

*Figure 1* Plots of $B$ of Equation 1 for (—) Rossi's stress concentration approach ($B \simeq 5a/4c + 3/4$) and (×) a minimum solid area approach (i.e. by equating Equations 1 and 2).

above are generally nearly identical with those calculated for minimum solid area models, i.e. the $b$ values of Equation 3 (hence also approximately those of Equation 4). Thus for the extreme of prolate spheroidal (i.e. cylindrical) pores parallel to the stress $b \simeq 1$, for spherical pores $b \simeq 3$, and for one limit of oblate spheroids (cylindrical pores whose axis is normal to the stress axis) $b \simeq 7$. Such $P$-independent $b$ values are close, but deviate somewhat from the calculated $P$-dependent $B$ values, especially for the extreme of oblate spheroids.

Consideration of another extreme of oblate spheroids, i.e. cracks aligned normal to the stress axis, casts further doubt on the correctness of Rossi's approach and the effects of local stress concentrations for features smaller than the failure-causing flaw size. For such oriented cracks his $B$ value is infinite, in marked contrast to recent theories for cracked materials. These typically follow Equation 1 with $B \simeq 4.6$ and 5, respectively, for slit for penny-shaped cracks aligned perpendicular to the stress and $P$ replaced by a crack density parameter which is equal to or greater than the net porosity volume of the cracks [22–23]. The disparity between Rossi's $B$ values and the effects of cracks is not restricted to $c/a = 0$. $P$ can be expressed as $(\pi/k)\ Na^3$, where $N$ = number of cracks or pores/volume, $a$ = crack or pore radius and $k$ depends on the $c/a$ ratio. For $c/a = 0.01$, 0.1, and 1, respectively, Rossi's B values are 5.27, 5.55, and 8.38, respectively, in contrast to $B \simeq 4.6$ to 5 for oriented cracks as noted above (or 1.6–1.8 for random cracks). Thus Rossi's $B$ values are too high, while those calculated based on minimum solid (i.e. bond) area (Fig. 1) are much closer to those for crack models, especially since lower $B$ values calculated at lower $P$ are pertinent for cracks. (Use of the average solid area gives $B = 1$ for all pore shapes, and agrees only with Rossi's and the minimum solid area for extreme prolate spherical, i.e. cylindrical, pores.)

## 4. Summary and conclusions

Evaluation shows that stress concentration effects from pore shapes provide little or no fundamental basis for determining elastic properties, generally not much more for tensile strength, but possibly more for mechanical properties involving significant compression loading. The approximate agreement of mechanical property–porosity models, especially elastic property models, based on stress concentrations from porosity with elastic data can be attributed to similar, but better, correlations between pore shape and minimum solid (load-bearing) area. Thus, minimum solid bond area is indicated as having more fundamental correlation with mechanical property–porosity behaviour.

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